this latter result. For this case, the peak negative overpressure magnitude is also presented to illustrate the importance of the negative overpressure region. As can be seen, consideration of both positive and negative overpressures results in a large decrease in peak negative overpressure magnitude with a relatively small penalty in peak positive overpressure.

Several assumptions were made following Eq. (6) that need some comment. The possible invalidity of $F(y)$ just behind the "corner" at $x = l$ is of no great concern since this region is cut off by the rear shock and only the integral of $F(y)$, which is given accurately by our approximate form, is needed. Our results show that the "corner" at $x = l$ is indeed convex for all σ_b, becoming smooth for $\sigma_b \rightarrow 0$, which confirms a posteriori our assumption. The assumption that, for the lower bound configuration, we should choose $y_1 = l$ at the rear shock is more difficult to justify in a simple manner. Suffice it to note that variational analysis indicates that $y_1 = l$ gives at least a relative bound.

The results presented above assume a uniform atmosphere. The effects of a stratified atmosphere on sonic boom have been treated by George and Plotkin[7] in a manner which can be easily applied to the previous results by a simple redefinition of the nondimensional variables σ_b and $\Delta\pi$. Using George's coefficients C_I, C_A, and C_T, if we redefine

$$\sigma_b = k\left(\frac{h}{l}\right)^{1/2}\frac{A_{e,b}}{l^2}\left(\frac{\gamma}{\gamma+1}\,\frac{\beta}{M}\,C_T\,\frac{c}{h^{1/2}}\right)$$

$$\Delta\pi = \beta^{-1/4}\left(\frac{h}{l}\right)^{3/4}\left(\frac{A_{e,b}}{l^2}\right)^{-1/2}\left|\frac{\Delta p}{p}\right| C_I C_A \times \left(\frac{h}{l}\right)^{1/2}\left(\frac{\gamma}{\gamma+1}\,\frac{\beta}{M}\,C_T\,\frac{c}{h^{1/2}}\right)^{1/2}$$

where c is the sonic speed at the altitude of the aircraft, the results presented in Figs. 1 and 2 are also valid for stratified atmospheres.

As an example, we show in Fig. 3 the lower bound equivalent area and F distributions for an SST aircraft 300 ft long and weighing 50,000 lb, flying at Mach 1.414 at 40,000 ft in a standard atmosphere. This corresponds to $\sigma_b = 0.3$ and has a peak overpressure magnitude of 0.74 psf on the ground (assuming a reflection coefficient of 2).

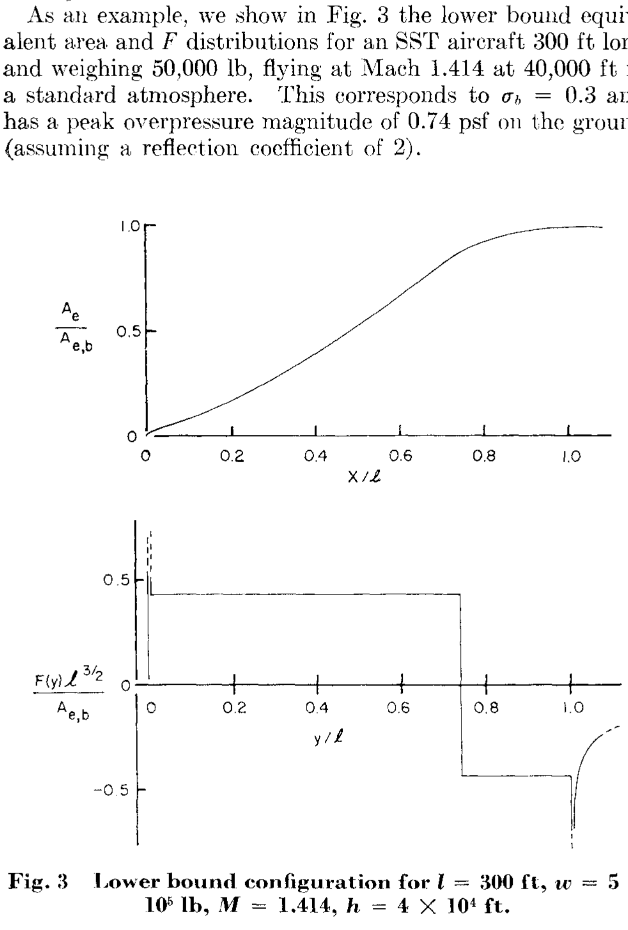

Fig. 3 Lower bound configuration for $l = 300$ ft, $w = 5 \times 10^5$ lb, $M = 1.414$, $h = 4 \times 10^4$ ft.

For comparison between the present results and those of Seebass, we consider a 600,000-lb aircraft which is 300 ft long and flying at 60,000 ft at Mach 2.7 in an atmosphere with a scale height of 20,000 ft. Our results show a peak overpressure magnitude of 1.19 psf compared with Seebass' result of 1.04 psf for the positive overpressure only (which corresponds to the lower curve in Fig. 2) and about 1.5 psf for his antisymmetrical signature. The present results indicate that, although it may be necessary to consider the negative overpressure region in configuration tailoring for sonic boom, the penalty incurred in doing so should not be large.

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Generation of Erroneous Line-of-Sight Rates by Radome Refraction Errors

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Introduction

A SIGNIFICANT measurement error for a radar system located in a tracking aircraft can be developed as a result of refraction of the beam as it passes through the radome. The time derivative of this refraction error creates, in effect, an erroneous line-of-sight rate of the target as seen by the radar. The purpose of this Note is to derive this error.

Analysis

The airborne radar carried in aircraft A tracks some target T, usually another aircraft or possibly a ground target. Designate the unit vector along the actual geometric line-of-sight (LOS) from A to T by $\bar{\imath}$. If $\dot{\bar{\imath}} \equiv (d\bar{\imath}/dt)$ is the derivative as seen in a nonrotating coordinate system centered at the interceptor, the true line-of-sight rate (LOSR) is

$$\bar{\Omega} = \bar{\imath} \times \dot{\bar{\imath}} \tag{1}$$

If the radome refraction error is δ, the apparent LOS lies along $\bar{\imath}'$ where $\bar{\imath}'$ is deflected from $\bar{\imath}$ by angle δ (Fig. 1).

From Fig. 1

$$\bar{\imath}' = \bar{\imath} + \Delta\bar{\imath} \tag{2}$$

where, since δ is small (e.g., less than 1°)

$$|\Delta\bar{\imath}| = \delta, \text{ in rads} \tag{3}$$

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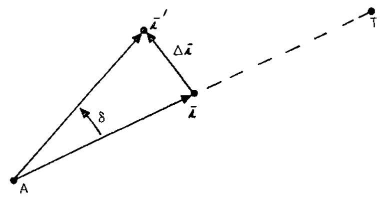

Fig. 1 Refraction error.

Now in accordance with Eq. (1), the erroneous LOSR is

$$\bar{\Omega}' = \bar{\imath}' \times \dot{\bar{\imath}}' \qquad (4)$$

Substituting Eq. (2) into (4)

$$\begin{aligned}\bar{\Omega}' &= (\bar{\imath} + \Delta\bar{\imath}) \times (\dot{\bar{\imath}} + \Delta\dot{\bar{\imath}}) \\ &= \bar{\Omega} + \Delta\bar{\imath} \times \dot{\bar{\imath}} + \bar{\imath} \times \Delta\dot{\bar{\imath}}\end{aligned} \qquad (5)$$

neglecting $\Delta\bar{\imath} \times \Delta\dot{\bar{\imath}}$ as second order. The error in the LOSR is

$$\Delta\bar{\Omega} = \bar{\Omega}' - \bar{\Omega} = \Delta\bar{\imath} \times \dot{\bar{\imath}} + \bar{\imath}\Delta \times \dot{\bar{\imath}} \qquad (6)$$

From Eq. (1), $\dot{\bar{\imath}} = \bar{\Omega} \times \bar{\imath}$, so the first term in Eq. (6) becomes

$$\Delta\bar{\imath} \times (\bar{\Omega} \times \bar{\imath}) = -\bar{\imath}(\Delta\bar{\imath}\cdot\bar{\Omega}), \text{ since } \Delta\bar{\imath}\cdot\bar{\imath} = 0 \qquad (7)$$

The term in Eq. (7) lies along $\bar{\imath}$ and hence cannot be tracked as an error in the LOSR. Now a vector $\bar{\delta}$ can be constructed perpendicular to $\bar{\imath}$ and such that

$$\Delta\bar{\imath} = \bar{\delta} \times \bar{\imath} \qquad (8)$$

The second term in Eq. (6) then becomes

$$\begin{aligned}\bar{\imath} \times \Delta\dot{\bar{\imath}} &= \bar{\imath} \times (\dot{\bar{\delta}} \times \bar{\imath} + \bar{\delta} x \dot{\bar{\imath}}) \\ &= \dot{\bar{\delta}} - \bar{\imath}(\dot{\bar{\delta}}\cdot\bar{\imath}), \text{ since } \bar{\imath}\cdot\dot{\bar{\imath}} = \bar{\imath}\cdot\bar{\delta} = 0\end{aligned} \qquad (9)$$

The right-hand side of Eq. (9) is the derivative $\dot{\bar{\delta}}$ less the component along $\bar{\imath}$. Hence, combining Eqs. (6) and (9)

$$\Delta\bar{\Omega} = (\dot{\bar{\delta}})_{\perp} \text{ to } \bar{\imath} \qquad (10)$$

Let y and z be the radar axes perpendicular to the LOS, then in general

$$\bar{\delta} = \bar{\jmath}\delta_y + \bar{k}\delta_z \qquad (11)$$

and

$$\dot{\bar{\delta}} = \dot{\bar{\jmath}}\delta_y + \bar{\jmath}\dot{\delta}_y + \dot{\bar{k}}\delta_z + \bar{k}\dot{\delta}_z \qquad (12)$$

where

$$\dot{\bar{\jmath}} = \bar{\omega} \times \bar{\jmath},\ \dot{\bar{k}} = \bar{\omega} \times \bar{k} \qquad (13)$$

and $\bar{\omega}$ is the rigid-body angular velocity of the tracking radar. Substituting Eq. (13) into (12) and suppressing the $\bar{\imath}$ component as required by Eq. (10)

$$\Delta\bar{\Omega} = \bar{\jmath}(\dot{\delta}_y - \omega_x\delta_z) + \bar{k}(\dot{\delta}_z + \omega_x\delta_y) \qquad (14)$$

where ω_x is the antenna roll rate around the LOS. Equation (14) gives the desired expression for the LOSR error generated by radome refraction errors. To carry the analysis further requires determination of $\dot{\delta}_y$, $\dot{\delta}_z$.

Now δ_y, δ_z are functions of the location on the radome where the beam pierces the radome. Typically, they are expressed as functions of two angular coordinates which determine the LOS attitude, e.g., azimuth and elevation E, A. Thus,

$$\delta_y = \delta_y(E,A), \qquad \delta_z = \delta_z(E,A) \qquad (15)$$

Then in Eq. (14)

$$\begin{aligned}\dot{\delta}_y &= (\partial\delta_y/\partial E)\dot{E} + (\partial\delta_y/\partial A)\dot{A} \\ \dot{\delta}_z &= (\partial\delta_z/\partial E)\dot{E} + (\partial\delta_z/\partial A)\dot{A}\end{aligned} \qquad (16)$$

The partial derivatives are called the radome error slopes. These derivatives must be measured for a particular radome, along with the refraction errors Eq. (15). Also needed in Eqs. (14) and (16) are the rates ω_x, $\dot{E}$, $\dot{A}$. These can be determined for a particular antenna gimballing arrangement as a function of the aircraft yaw, pitch and roll rates and the antenna rates about the y and z axes.

Conclusions

An expression for the vector LOSR error generated by radome refraction errors and their slopes is given by Eq. (14). What is needed are measured or assumed refraction errors and their slopes as a function of radome location, and various rates such as $\dot{E}$, $\dot{A}$, and ω_x. Numerical solution generally requires a computer. These expressions have formed the basis for studying the effects of radome errors in airborne tracking systems.

A Digital Computer Study of System Modeling by Pulse Testing

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THE objective of this Note is to report on a method to reduce the data obtained from a pulse test by digital computer methods and to approximate the data functionals by Fourier series in order to facilitate Fourier transform calculations.

The gathering of information about the dynamic properties of physical systems constitutes a problem for eng neers in all fields. For systems or components with low natural frequencies or short test times, such as aircraft, sinusoidal testing to obtain dynamic data is too lengthy and very costly. For these systems the best test input is usually a transient or pulse input. These inputs are defined as identically zero for all values of time less than zero and time greater than some finite value. Therefore, the system output wil also die out in finite time. Only one test needs to be run and it usually can be done quickly. Many investigators[1–4] have reported on the difficulties that arise from the processing of physical data taken from pulse tests of aircraft performance. One basic difficulty was found by Lees[5] to be nonconvergence of the numerically (digitally) calculated Fourier transforms (transfer functions) from the sampled data of the input-output time histories. Various interpolating polynomials were used to obtain integrals of the time histories of the input and output. Stepped function, trapezoidal, and parabolic approximations were attempted with the same nonconverging result. He noted a 5% accuracy of the transfer function calculation at a frequency of only 5% of the sampling theorem limit. He also notes the classical "breakdown" phenomenon that usually is present at high

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